NATIONAL BUREAU OF STANDARDS REPORT

2355

A HISTORICAL NOTE ON THE APPLICATION OF THE "WEAKEST-LINK" IDEA TO TENSILE STRENGTHS

by

Julius Lieblein

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U. S. DEPARTMENT OF COMMERCE
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- Office of Basic Instrumentation
- Office of Weights and Measures.

Errata

for

National Bureau of Standards Report 2355

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Page	Line	Now reads	Should read
1	7 from bottom	80	70
4	1	S_x is	S_x is [sic — the author meant S_n
6	6	article	1880 article
8	6 from bottom	than short	than short ones

NATIONAL BUREAU OF STANDARDS REPORT

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A Historical Note on the Application
of the "Weakest-Link" Idea to Tensile Strengths

by

Julius Lieblein*

ABSTRACT

It appears to be commonly believed that the use of the "weakest-link" hypothesis and extreme value methods in connection with strength of test specimens originated with F. T. Peirce in an article published in 1926.

The discovery by the writer of a pair of long-forgotten articles in two engineering journals of the 1880's shows that we must push the date of first application of extreme values back by nearly 50 years, if not more. These articles also show a surprisingly modern approach, and it is therefore felt worthwhile to discuss them to some extent, particularly since they may not be readily accessible.

* National Bureau of Standards, Washington, D. C.

INTRODUCTION

Practically every writer (e.g. Epstein [1]) who makes a historical reference to the use of extreme values in tensile strength testing dates such application from Peirce's paper in 1926 [2], which in turn is based on Griffith's theory of flaws enunciated in 1920 [3]. In the interests of historical accuracy, it therefore seems important to report any material that comes to light which would significantly affect the widely-held belief concerning priority. The purpose of this note is to discuss excerpts from two articles written by W. S. Chaplin, Professor of Civil Engineering, University of Tokyo, that appeared in American engineering journals over 80 years ago and recently came to the attention of the writer.[1] The first [4] was published in Van Nostrand's Engineering Magazine for December 1880 and was entitled "The Relation Between the Tensile Strengths of Long and Short Bars;" the second [5] appeared in Proceedings of the Engineers' Club for 1882 and was entitled "On the Relative Tensile Strengths of Long and Short Bars."

1/ The writer is indebted to Dr. Churchill Eisenhart for showing him an old engineering text [6] in which these articles were cited.

CHAPLIN'S ARTICLE OF 1880

This was a brief article which explained the simple theoretical points involved in the "weakest-link" procedure. The author began by mentioning the applicability of the (Gaussian) "law of errors" to the variation in strengths of <u>individual</u> specimens or pieces of a given size, and showed that p^n is the chance that each of n pieces of the same size would exceed a certain tensile strength if p is the probability that each piece exceeds it. He then considered the key question that was the purpose of the article (p. 442):

> " Suppose that many pieces of cross section c, and length one inch have been tested for tensile strength with an average result S_o, and a probable variation in one piece of P_o; what will be the probable average strength, S_n, of pieces of the same cross section and a length of n inches?

" Knowing the probable variation in a piece one inch long, we are able to construct the curve showing the probability of any and all variations in a piece of this length. From this curve we can obtain the probability that the piece one inch long will break between any limits of variation. The probability that an inch-piece will break above a negative variation $-x$ is $.5 + A_x$, in which A_x represents the probability that the piece will break between 0 and $-x$. In a piece n inches long there are n pieces one inch long; the probability that any one of these will break above $-x$ being $.5 + A_x$, the probability that all of them will break above this limit, or that the <u>strength of the whole piece will be at least S_o</u> $-x$, [emphasis supplied] will be

$$(.5+A_x)^n \quad .$$

As S_x is an average. it is as probable that a piece n inches long will break above it as below it; hence the probability that a piece n inches long will break above it is .5. We have then

$$(.5+A_x)^n = .5 ,$$

in which A_x is the unknown quantity. We easily obtain

$$A_x = \sqrt[n]{.5} - .5; \text{ or } 2A_x = 2(\sqrt[n]{.5} - .5)."$$

In (not very different) modern symbols and terminology this stated essentially that if $\Phi(x)$ is the (Gaussian) probability that the tensile strength of a piece of given length is greater than x, then the probability that a piece n times as long has strength greater than y is $[\Phi(y)]^n$. Putting $[\Phi(y)]^n = \frac{1}{2}$ and using the appropriate normal tables yields the average (i.e. median) value of the strength y of a piece n inches long (of similar cross-section).

The underlined portion above is the essence of the "weakest-link" theory— that a chain is no stronger than its weakest link. It is rather interesting that, unlike

we of today, the author managed to express the idea quite clearly without the use of this overworked phrase.

The author then converted the relation $[\Phi(y)]^n = \frac{1}{2}$ into a useful table and chart, and explained that if $\frac{1}{2}$ were replaced by 1/4 in this relation the corresponding value of y would give the probable error for variation in strengths of the long specimen. This theory was then compared with the results of tests on 5 specimens of annealed Japanese copper wire of lengths 1, 4, 8, 12, 16 inches, and agreement of theory with experiment was found to be very satisfactory. Other tests were cited, and one of these sources [7] provided a clue for a still earlier reference (1871) to the idea of a material breaking at its weakest part. The entire passage involved is interesting, and is here quoted in full (emphasis supplied):

> " The idea was at one time prevalent, that the tensile strength of a bar of iron, etc., became greater by the bars being broken; for it was found that a piece of a bar broken by tension would bear more than the entire bar did; and that if a piece of a piece were broken

> in the same way, it would in turn support more than before. <u>But this proved nothing more than that the bar broke at its weakest part first,</u> and at the next weakest in succession."

The article concluded (p. 444) with a note of advice that is not without interest in the light of modern experimental procedures:

> " It is to be hoped that those who have testing machines and occasion to make numerous tests will publish either all their individual results, or will give the probable variation as well as the average strength of the materials which they study. It really tells but little about a material to give only the average breaking weight; uniformity of strength, or a small probable variation, is a very valuable quality, and without knowing whether a material has a small probable variation or not, no engineer can properly decide what factor of safety shall be used in designing a structure."

CHAPLIN'S ARTICLE OF 1882

This later paper, which went into greater detail than the one in 1880, had for its purposes (p. 18):

> "1. To prove that long bars are on the average weaker to resist tensile stress than short ones of the same material and cross-section;
>
> "2. To show how the reduction in strength may be found when the proper experiments have been made on the short bars."

The author's first point was carried out in a few brief paragraphs. After explaining that iron is not a uniform material and that tensile strength must vary from bar to bar, he stated (p. 19):

> " A bar under tensile stress yields and breaks at one point; it is more or less extended at all points, but it breaks at only one, and it is the strength at this point which determines the strength of the bar to resist tensile stress.

" Suppose now that pieces a, a_1, a_2, etc., have been cut from a bar A, and tested in the usual way. The average strength of the pieces a, a_1, a_2, etc., will necessarily be greater than the strength of the weakest piece; or, as <u>it is the weakest piece which determines the strength of A</u>, [emphasis supplied] the average tensile strength of the pieces a, a_1, a_2, etc., will be greater than the strength of A. Or <u>long bars are on the average weaker to resist tensile stress than short of the same material and cross-section</u>. It would have seemed unncessary to point out and insist on this fact, if it had not been denied by as great an authority as Fairbairn."

Apparently this doctrine of the "weakest piece" must have been promulgated sometime earlier[2] in order for it to have been considered with sufficient attention to elicit a denial by a top worker in the field.

The author's line of reasoning concerning his second point, calculation of the reduction in strength due to length, was essentially an elaboration of his method in the first article. It is also interesting to note his lament on the paucity of experimental data of the kind necessary for testing the theory. He indicated that his theory would have important practical application in showing that the usual factor of safety was always an overstimate. Thus for the Niagara suspension bridge, he found that the safety factor of 3.07 should really be only 2.76, a reduction of 10 percent.

In spite of the promising outlook for the theory the article seems to have ended on a pessimistic note and a hope that is still far from being realized (p. 28):

2/ Other evidence for this is provided by the quotation from [7] given above.

" Although I do not hope to see any practical application of this law, yet I consider it of some importance as one of those steps which will finally give us such a knowledge of the strength of materials that engineers will no longer be obliged, as they all now are, to make structures much stronger in practice than they are in theory."

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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